Game Theory in Economics

Advanced article

Kevin A McCabe, George Mason University, Fairfax, Virginia, USA

CONTENTS

Game theory is an increasingly important tool to help economists understand the strategic interaction between groups of individual decision makers.

INTRODUCTION

One of the most frequent observations in experimental economics is that there is a great deal of variation in how subjects play. It seems reasonable to hypothesize that this variation exists because individuals differ in how they think about other players' rationality. While game theory provides us with a wealth of models that help provide retrospective explanations of why this variation is observed, these models have far less predictive capability. What seems to be missing is a unified treatment of cognition in strategic settings.

GAME THEORY AND DECISION THEORY

Game theory can be seen as the natural extension of rational-choice models of individual decision making as modeled by decision trees (Luce and Raiffa, 1957). Figure 1 gives a simple example of a decision tree. The first and second nodes of the decision tree are decision nodes, the first belonging to player 1, and the second to Nature. Player 1 must decide, before knowing Nature's move, whether or not to stop and make \$10 for sure, or go on and face a probability p of getting \$15 and $1-p$ of getting \$0. If player 1 desires to maximize the expected pay-off, then the decision whether or not to go on depends on whether p is sufficiently large that $p \times 15 + (1-p) \times 0 \geq 10$. If $p \geq 2/3$ then player 1 should go on.

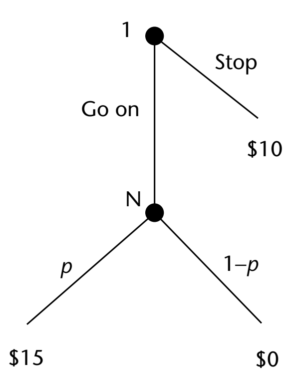

Figure 1. An individual decision problem for player 1 with perfect information.

We have assumed that the individual is interested in the monetary pay-offs associated with the outcome (or terminal) nodes. The usual economic assumption is that, everything else being equal, our player will prefer more money to less. However, in Figure 1 our player must choose between \$10 for sure, or a gamble g that pays \$15 with probability p and \$0 with probability $1-p$. We have assumed that our player will compute the expected value of g, that is, $15p$, in order to compare the value of the gamble to \$10. More generally, we could use expected-utility theory to model how our individual will choose between \$10 and the gamble. Let $U(x)$ be the subjective value that our player places on \$$x$. We assume U is an increasing, but not necessarily linear, function. In expected-utility theory, the value of g is calculated as $\mathrm{E}U(g) = pU(15) + (1-p)U(0)$, and to make a choice our player would compare $U(10)$ to $\mathrm{E}U(g)$. If the subject is trying to maximize expected utility, then the utility values should replace the pay-offs shown in Figure 1.

If the utility function for a player is nonlinear, an experimenter who wants to control individual values in an experiment faces a difficulty, since ultimately the experimenter pays off in dollars, not utilities. One approach is to risk-neutralize a subject by inducing a linear utility function. To do this, give the subject only two possible pay-offs, say \$5 and \$25. Now replace each dollar of pay-off with a red lottery ticket. For example, \$0 is replaced by 0 red tickets, \$10 is replaced by 10 red tickets, and \$15 is replaced by 15 red tickets. When an outcome node is reached, the subject earns some number of red lottery tickets. Blue lottery tickets are then added until the total number of tickets is equal to 15, and all the tickets are then placed in an urn. One ticket (red or blue) is then randomly drawn from the urn. If the ticket is red, the subject earns \$25, valued at $U(25)$, and if it is blue, the subject earns \$5, valued at $U(5)$. Now, a subject who stops earns 10 tickets, valued at $\frac{1}{3}U(5)+\frac{2}{3}U(25)$. A subject who goes on earns either 15 tickets, valued at $U(25)$, with probability p, or 0 tickets, valued at $U(5)$, with probability $1-p$. We then know that the subject should go on just when $p \geq \frac{2}{3}$ since this is exactly the condition for $pU(25)+(1-p)U(5) \geq \frac{1}{3}U(5)+\frac{2}{3}U(25)$.

From here on we will simply use pay-off numbers, which can be thought of as utility values, but in experiments these numbers are either dollars or lottery tickets.

Consider the decision problem in Figure 2. Nature moves twice, but our player does not know which branch Nature chooses initially. If Nature has chosen the left branch, then Nature will be more likely ($p=\frac{3}{4}$) to choose 15 for the player, but if Nature has chosen the right branch, then Nature will be less likely ($p=\frac{1}{4}$) to choose 15 for the player. In the figure the decision nodes for the player are connected by a dashed line, indicating that they are in the same information set for the player. Nodes in the same information set must have the same number of branches, and they must all be played in the same way by the player, since the player does not know which of the decision nodes he or she is at. Suppose the player believes that Nature will choose the left branch 90% of the time, i.e., $q=0.9$. If the player decides to stop, the player gets 10, but if the player decides to go on, the expected pay-off is $0.9\times\frac{3}{4}\times 15+0.1\times\frac{1}{4}\times 15=10.5$. Since $10.5>10$, player 1 should go on.

EXTENSIVE-FORM GAMES

In Figure 1, Nature does not get a pay-off, or make choices, but instead is regarded as a probability distribution over possible actions outside the control of the individual. The use of types is suggested by Harsanyi (1995). If we replace Nature with another individual, we get the game tree shown in Figure 3. The pay-offs to player 2 are written below those to player 1. Thus, if player 1 stops, then player 2 also gets 10, but if player 1 goes on, then player 2 can either move left, in which case player 2 gets 25, or right, in which case player 2 gets 40. Given that player 2's actions are still outside the control of player 1, how should player 1 decide? To answer this question we can analyze the new decision problem as a two-person extensive-form game.

An extensive-form game with complete and perfect information can be defined as follows. Define a 'branch' as an arc that connects two nodes. A branch represents a move in the game. A node is an 'initial' node if it has no branches going into it, but has at least one branch leaving it. A node is a 'terminal' node if it has at least one branch going into it but no branches leaving it. A 'path' is a sequence of nodes that are connected by branches, starting at an initial node and ending at a terminal node. Nodes and branches must satisfy the following properties:

- All decision nodes are played by only one player.
- Each terminal node has a pay-off for every player in the game.

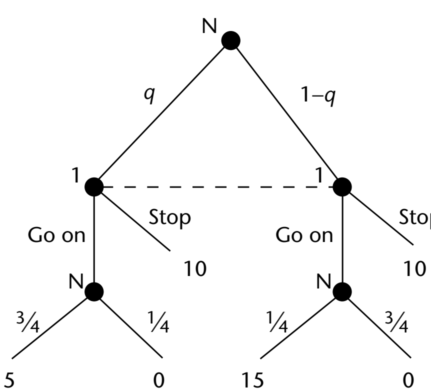

Figure 2. An individual decision problem for player 1 with imperfect information.

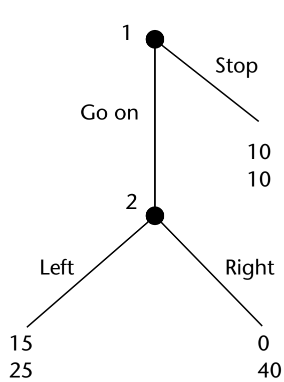

Figure 3. A two-person decision problem with perfect information.

- There is only one initial node.
- Every node, except the initial node, is connected to some earlier node (called its 'predecessor') by a single branch.
- Every terminal node is connected to the initial node by a unique path.

Kuhn (1953) showed that the principle of backwards induction can be used to guide player decisions. In backwards induction, each player reasons backwards through the tree, starting with the terminal nodes, and making the rational choice for each player at each decision node, assuming that decisions made after that node will also be rational.

The decision problem shown in Figure 3 is an extensive-form game with complete and perfect information. Using backwards induction, player 1 should reason as follows. If player 2 gets to move, player 2 will move right, since $40 > 25$. Therefore player 1 should stop.

However, in experiments with cash-motivated subjects, about half of the player 1s choose to go on, and of those player 2s who get to move, over two-thirds reciprocate the favor by moving left.

How can we explain this divergence from the theory? The value of game theory as a modeling tool becomes apparent as the experimenter attempts to reconcile the differences between theory and observation. One approach to explaining this divergence is to assume that Nature produces different types of player 2s.

Consider a simple model in which Nature creates two types of individuals who may end up being player 2. A 'trustworthy' type of player feels strongly compelled to reciprocate when believing she has received a favor. Moving left as player 2 may result in a pay-off of 40, but player 2 may 'feel' far worse off, like getting a pay-off of only 20. Trustworthy players will therefore move left, because $25 > 20$. The second type for player 2, the 'me' type of player, feels no such compunction, and plays right, since $40 > 25$.

This game can be described as in Figure 4, with Nature moving first. Notice the addition of an information set as a dashed line connecting player 1's two decision nodes. Player 1 does not know which node he is at, that is, which type player 2 is, when it is player 1's turn to move. On the other hand, player 2 knows his or her own type when player 2 gets to move.

Now suppose player 1 believes that more than two-thirds of player 2s provided by Nature are of the trustworthy type. (This frequency is shown as q in Figure 4.) Then player 1's expected pay-off is higher by going on, since $q \times 15 + (1 - q) \times 0 > 10$.

STRATEGIC-FORM GAMES

The extensive-form game shown in Figure 3 can be rewritten as the 'normal-form' or 'strategic-form' game shown in Figure 5. While in extensive-form games the order of moves is important, in a strategic-form game players choose their strategies simultaneously. All normal-form games can be written as n-dimensional arrays, with n being the number of players, making the game simpler to analyze. Kohlberg and Mertens (1986) show that the essential strategic features of all extensive-form games are retained when the games are converted to strategic form. When players move simultaneously it is less clear how they should think about the other player. Consequently, from a cognitive viewpoint it is likely that subjects think differently about how to play an extensive-form game and a strategically equivalent strategic-form game.

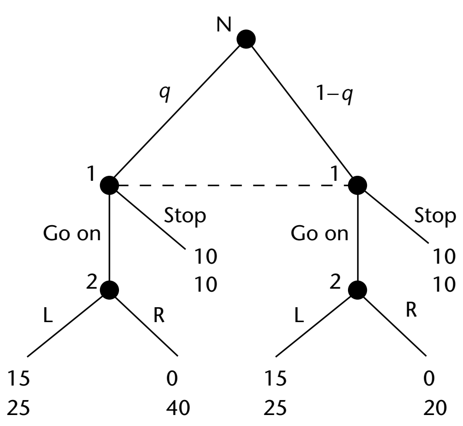

Figure 4. A two-person decision problem with incomplete information.

		Player 2: Left	Player 2: Right
Player 1	Stop	10 10	10 10
	Go on	15 25	0 40

Figure 5. A two-person decision problem as a normal-form game.

In analyzing a strategic game, players must put themselves in each other's shoes and try to reason about how the other player will reason. For example, in Figure 5, player 1 can notice that player 2 is always no worse off by moving right. Such a choice is called a 'weakly dominant' strategy for player 2. Alternatively, we can say that moving right is a best response for player 2 independently of player 1's choice. For player 1, stopping is a best response to player 2's decision to move right. The pair of pure (deterministic) strategies (Stop, Right) is thus a 'Nash equilibrium', defined as a strategy for each player that is a best response to all the others.

Not every strategic-form game has a pure-strategy Nash equilibrium: consider, for example, the popular children's game of rock–scissors–paper, shown in Figure 6. If we extend our concept of a strategy to include 'mixed strategies' – choices of probability distributions over pure strategies – then, as Nash proved, every strategic game has a Nash equilibrium. In the case of rock–scissors–paper, each player should play each strategy with equal probability; i.e., the Nash equilibrium strategy is (⅓, ⅓, ⅓).

However, if player 2 knows that player 1 will play (⅓, ⅓, ⅓), then player 2 is just as well off playing (1, 0, 0), or rock for sure. Of course, player 2 does not want player 1 to know this, but how will player 1 find out? One approach to solving the 'incentive problem' associated with mixed strategies is to view a mixed strategy as the frequency distribution of pure strategies played in a frequently-replayed perturbed version of the original game (Harsanyi, 1977). Imagine that a game is frequently occurring and that a large population of potential players keeps experiencing minor fluctuations in their pay-offs. On average, they expect to play against someone playing (⅓, ⅓, ⅓). But suppose that if rock wins player 2's payoff is now $1 + \varepsilon$, where $\varepsilon > 0$ is the small perturbation. Clearly player 2 now prefers (1, 0, 0), and will play it.

Figure 7 illustrates a game with two Nash equilibria, (Up, Left) and (Down, Right). What makes this game interesting is that even though (Up, Left) is preferred by both players, coordinating strategies on this equilibrium in simultaneous play is made difficult by the degree of risk associated with being wrong about guessing what the other player will do. Player 1 may reason as follows. 'If I play Up, and player 2 plays Right, then I get my worst pay-off, 0, and player 2 gets 9. Since getting 9 is not much worse than getting 10, player 2 has not given up much by playing Right, and if I play Down, then player 2 still gets 5. So player 2 has a lot of reasons to play Right. What is worse is that player 2 has probably reasoned the same way about me and is now convinced I will play Down, which I better now do.' Harsanyi and Selten (1988) appeal to the 'principle of insufficient reason' to define an equilibrium as 'risk-dominant' if each equilibrium strategy is a best response to the $n - 1$ other-players mixed strategies that put equal weight on each of their possible pure strategies.

RATIONALIZING OTHER PLAYERS' DECISIONS

An important question in game theory is: what do people need to know about each other in order to make rational strategic choices? We have mentioned the 'default' reasoning, the principle of insufficient reason, which gives equal weight to other players' strategies. However, before falling back on this principle, subjects may do better to apply other methods such as backwards induction. Backwards induction can be generalized to strategic-form games as follows.

Player 1 \ Player 2	Rock	Scissors	Paper
Rock	0 0	1 0	0 1
Scissors	0 1	0 0	1 0
Paper	1 0	0 1	0 0

Figure 6. The game of rock–scissors–paper.

Player 1 \ Player 2	Left	Right
Up	10 10	0 9
Down	9 0	5 5

Figure 7. A risky coordination problem.

We say that a player's strategy s_i, is 'strictly dominated' by another strategy s_j if playing the strategy s_j will always return a higher pay-off than playing the strategy s_i. In this case, there is no set of beliefs for this player about the other players that would rationalize the use of the strategy s_i, and therefore we can eliminate this strategy from the player's choices. We can then repeat this procedure, eliminating any strictly-dominated strategies that remain, until all such strategies are gone. This process of iterative elimination of strictly-dominated strategies leaves only those strategies that are 'rationalizable'.

This principle can be generalized (Bicchieri, 1993; Osborne and Rubenstein, 1994). For example, we can extend this procedure to also eliminate weakly-dominated strategies. In Figure 5, Right weakly dominates Left for player 2 since it makes player 2 no worse off, no matter what player 1 does. Therefore we should eliminate Left. But then, for player 1 Stop strictly dominates Go-on, so we should eliminate Go-on. What remains is the dominance-solvable pair (Stop, Right).

Another principle that players may use is the principle of forward induction, as illustrated in Figure 8 – an extension of the game shown in Figure 4. In this game, after Nature moves by choosing player 2's type, player 2 gets to move. Player 2 can opt out and get 30, or player 2 can continue. The principle of forward induction asks: what should player 1 assume about player 2's strategy if player 2 chooses to continue? Obviously, player 2 is trying to win more than 30, which can occur only if Nature has chosen a 'me' type of

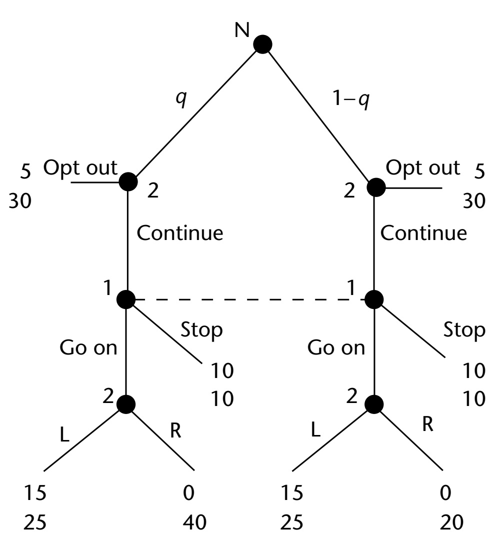

Figure 8. A two-person decision problem with incomplete information.

player on the left branch. So, if player 2 chooses to continue, player 1 should realize that player 2 plans to move right, and therefore player 1 should stop. But this means that player 2 should always opt out, leaving player 1 with 5.

Going back to the risky coordination game in Figure 7, suppose we add the proviso that player 1 gets to choose whether or not to opt out and get 6 or continue and play the game. Again using the principle of forward induction, player 2 should assume that by choosing to continue player 1 plans to play Up. Furthermore, player 1 should assume that player 2, having made this analysis, will play Left. Thus player 1 should choose Continue, and then play Up with greater confidence that he or she will end up with 10.

REPEATED GAMES

Repeating a game allows for strategy spaces in which the play in future games depends on the play in past games. For example, in Figure 5, repeat game strategies (such as 'tit-for-tat' in the prisoner's-dilemma game) may be useful in achieving the more efficient (15, 25) or (0, 40) outcomes rather than the (10, 10) outcomes. Suppose, for example, player 1 adopts the following 'trigger' strategy: start by playing Go-on, and continue to play Go-on as long as player 2 continues to play Left; if player 2 chooses to play Right, then play Stop from then on. By playing Right, player 2 can get 40 (instead of 25), but from then on player 2 will get only 10.

If there is a fixed number of periods, this strategy is not an equilibrium strategy, for the following reason. In the last period, player 2 should play Right. Player 1 should realize this and play Stop. But then, player 2 should play Right in the second-to-last period. Again, player 1 should realize this and play Stop. By backwards induction, cooperation is unraveled all the way to the first period, with player 1 playing Stop. Kreps *et al.* (1982) show how adding a little bit of uncertainty about the other player's type is sufficient to achieve cooperation in the finitely-repeated prisoner's-dilemma game. Axelrod (1984) has shown that in practice people don't unravel the game using backwards induction, but rather play relatively myopically, while still incorporating elements of the repeat-game strategy. Given this, the trigger strategy can be replaced by the more forgiving strategy tit-for-tat, resulting in even larger efficiency gains.

McCabe and Smith (2002) report on a series of experiments with properties similar to the game shown in Figure 3, but where they vary the likelihood that people will play each other again. They find that the likelihood of meeting again influences subjects' levels of cooperation. Both theoretical and empirical results point to the need for the experimenter to control for this likelihood. Different protocols are possible for an experiment with four people in which no pair will play each other twice. In the 'everyone meets once' condition, a total of six games will take place. However, there is still room for an indirect effect. In the first period, subjects 1 and 2 play and subjects 3 and 4 play. In the second period, subjects 1 and 3 play and subjects 2 and 4 play. In the third period, subjects 1 and 4 play and subjects 2 and 3 play. But subject 1 has already played subject 2 who has already played subject 4, so it is possible that subject 1 has already influenced subject 4's behavior before they play. To eliminate this contagion effect, the 'turnpike matching' condition is employed. This control reduces the number of observation pairs to four. In the first period, subjects 1 and 4 play. In the second period, subjects 1 and 3 play and subjects 2 and 4 play. In the third period, subjects 2 and 3 play. So subject 1 meets subject 4 and then subject 3, and subject 2 meets subject 4 and then subject 3.

References

Axelrod R (1984) *The Evolution of Cooperation*. New York, NY: Basic Books.

Bicchieri C (1993) *Rationality and Coordination*. Cambridge, UK: Cambridge University Press.

Harsanyi J (1977) *Rational Behavior and Bargaining Equilibrium in Games and Social Situations*. Cambridge, UK: Cambridge University Press.

Harsanyi J (1995) Games with incomplete information. *American Economic Review* **85**: 291–303.

Harsanyi J and Selten R (1988) *A General Theory of Equilibrium Selection in Games*. Cambridge, MA: MIT Press.

Kohlberg E and Mertens J-F (1986) On the strategic stability of equilibria. *Econometrica* **54**: 1003–1037.

Kreps D, Milgrom P, Roberts J and Wilson R (1982) Rational cooperation in the finitely repeated prisoners' dilemma. *Journal of Economic Theory* **27**: 245–252.

Kuhn H (1953) Extensive games and the problem of information. In: Kuhn HW and Tucker AW (eds) *Contributions to the Theory of Games*, pp. 193–216. Princeton, NJ: Princeton University Press.

Luce D and Raiffa H (1957) *Games and Decisions*. New York, NY: John Wiley.

McCabe K and Smith V (2002) Strategic analysis by players in some games: what information do they use? In: Ostrom E (ed.) *Trust and Reciprocity*. New York: Russell Sage Foundation.

Osborne M and Rubinstein A (1994) *A Course in Game Theory*. Cambridge, MA: MIT Press.

Further Reading

Gintis H (2000) *Game Theory Evolving*. Princeton, NJ: Princeton University Press. [An introduction to many of the recent advances in game theory along with numerous interesting examples.]

Nash J (1950) Equilibrium points in *n*-person games. *Proceedings of the National Academy of Sciences of the USA* **36**: 48–49.

von Neumann J and Morgenstern O (1944) *Theory of Games and Economic Behavior*. Princeton, NJ: Princeton University Press.

Smith V (1992) Game theory and experimental economics: beginnings and early influences. In: Weintraub ER (ed.) *Toward a History of Game Theory*. Annual Supplement to *History of Political Economy* **24**: 241–282. [A historical account of the early influences of game theory on experimental economics.]